
SPEA-V348 Management Science · Week 5 review

Decision trees, and sequential decisions

Both sessions, every slide, fully revealed. Read it without me in the room.

Mon Sep 21 and Wed Sep 23, 2026

The week at a glance

Mon Sep 21	The stockpile table drawn as a tree and folded back from the right, with one rule for each shape
Wed Sep 23	Decide, observe, decide again: the strategy, the value of the second decision, the flip probability, and EVPI on a tree
Reading	Taylor 13e, Chapter 12, pp. 587 to 596, for the method; ignore its add-ins
Due	The Week 5 workbook and a picture of your tree from the tree builder, on Canvas. Tier 1: AI as a tool, with disclosure

ON THE SITE · the Week 5 page

The widgets, both self-checks, the tree builder, and the workbook.

v348.jima.me/week/w05/



Session 1

Mon Sep 21

Why draw it at all

A table holds one decision; a tree holds a sequence

A payoff table

one decision, one uncertain event, one payoff per cell



A decision tree

the same cells laid out in the order things happen, with room for a decision that waits on an outcome

For a one-shot decision the two are the same object and give the same answer. The tree earns its keep the moment a **second decision** depends on something you cannot see yet.

Today: draw the table as a tree and fold it back. Wednesday: give the tree a second square.

The outbreak stockpile

The question this session answers

PUZZLE

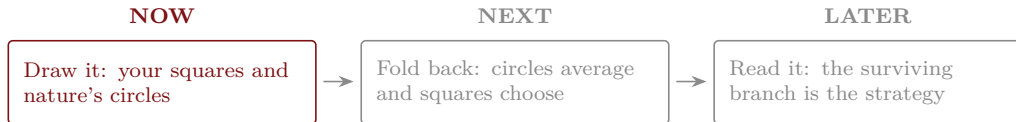
A county health office must buy protective supply before flu season: **stockpile big**, **stockpile small**, or **wait and see**, not knowing whether the season will be severe or mild. The epidemiologists say **Severe 0.4, Mild 0.6**. Net benefit, in \$ millions:

Response	Severe (0.4)	Mild (0.6)
Stockpile big	120	-30
Stockpile small	70	40
Wait and see	-50	90

Which choice, and how do you draw the whole decision so a board can follow it?

Where we are

Three blocks today, in this order



The first block weights the county's payoffs by their probabilities, then draws the table as a tree. "So a board can follow it" is the other two: fold the tree back to the number, then read the plan off the branches that survive.

Weight each stockpile payoff by its chance

Severe 0.4, Mild 0.6; net benefit in \$ millions

Choice	Probability-weighted payoffs	EV
Stockpile big	$0.4(120) + 0.6(-30)$	30
Stockpile small	$0.4(70) + 0.6(40)$	52
Wait and see	$0.4(-50) + 0.6(90)$	34

*Stockpile small has the highest EV, **\$52M**. EV needs probabilities; maximax, maximin, and minimax regret compare payoffs without them.*

Three shapes, and only three

Everything on a decision tree is one of these

A square is a decision node: a fork *you* control. Which stockpile to buy

A circle is a probability node, Taylor's term; most people say chance node. A fork *nature* controls, with a probability on every branch

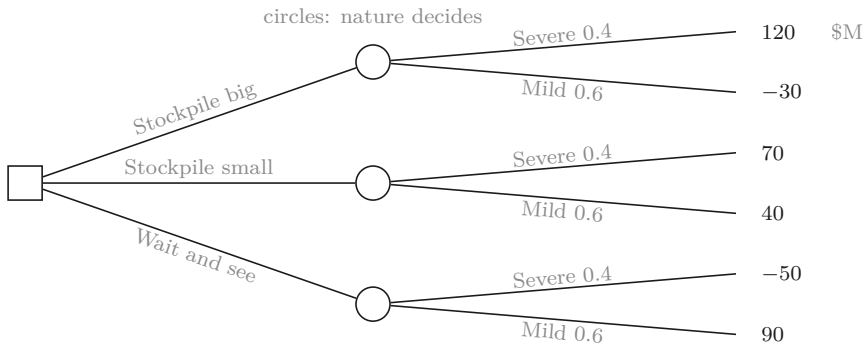
A branch is one alternative out of a square or one state out of a circle. A branch that goes no further ends in a *leaf*: the payoff

*Squares for what you decide, circles for what you cannot, and the whole thing read **left to right in the order things happen.***

Taylor 13e, p.587.

The stockpile, as a tree

One square, three branches, a circle on each, six leaves



The square first, because it is decided first. The circles come next because the season is learned after the order is placed, and every leaf is a cell of the table.

Three drawing habits

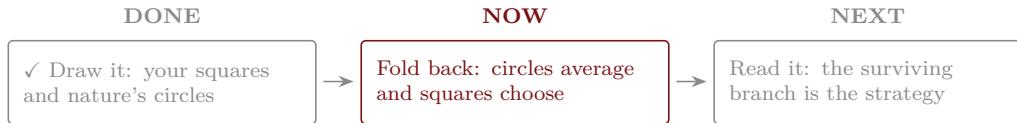
They keep a tree honest, and they are graded

- **Time runs left to right.** The first thing decided is the leftmost square. Anything learned later sits to its right.
- **Every branch out of a circle carries its probability**, and the branches out of one circle sum to 1: $0.4 + 0.6$, because they cover every way that one event can resolve.
- **A cost sits on the branch where it is paid**, in parentheses. Taylor subtracts it once, at that branch; netted out of every leaf instead, a cost shared by several leaves is easy to count twice.

A tree that breaks one of these folds back to the wrong number, and looks fine while it does.

Where we are

The tree is drawn, two blocks to go



You have one square, three circles, and six leaves, every leaf a cell of the table and every circle's 0.4 and 0.6 summing to 1. The tree does not yet say which branch to take: that needs a rule for each shape.

Fold it back: two rules, one for each shape

Start at the right and work toward the first square

At a circle, average. Multiply each branch's payoff by its probability and add: its expected value. You do not choose which branch happens

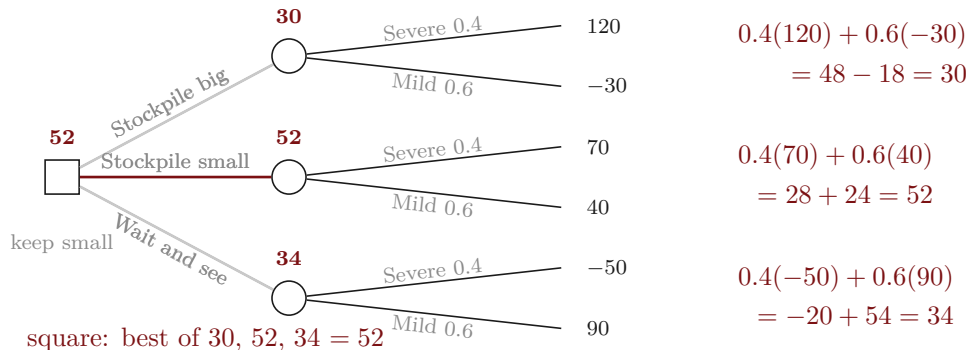
At a square, keep the best. Take the branch worth most and prune the rest. You do choose, and you will never take a worse branch when a better one is open

*The mistake of the week is **averaging at a square**. It prices you as though you chose at random, and a board never does.*

Taylor 13e, pp. 588 to 589: the expected value is computed at each probability node, and the branch with the greatest expected value is selected.

Fold the stockpile back

Circles: weight each payoff by its probability and add. The square: keep the best. Values in \$M



Week 5 page, outbreak stockpile: each circle is a probability-weighted average. The square keeps the largest, 52, and prunes the other two.

Fold it back on your laptop

IN CLASS

Open the tree widget on the Week 5 page and press **Fold back**. Watch the order: the three circles light up first, and only then does the square keep its best branch and grey out the other two. Press **Reset** and run it again, saying the rule out loud at each shape before the widget applies it.

ON THE SITE · the tree widget

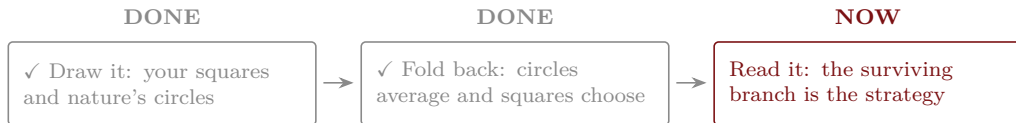
The stockpile tree, with the two fold-back rules written out just below it.

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Where we are

Drawn and folded, one block to go



The circles roll up to 30, 52, and 34, and the square keeps the 52, matching the payoff table's expected values. What a board reads off the tree is not the 52 but the branch that survived.

Reading the answer off the tree

- The surviving branch at every square, read left to right, is the **decision strategy**: the plan the tree recommends.
- With one square the strategy is one word, **stockpile small**, worth \$52M. Wednesday's trees have a second square, and the strategy becomes a sentence with an "if" in it.

A tree is answered by the branches that survive, not by the number at the root alone.

Software that draws and folds

The course's own tree builder, once the hand method is yours

- The **tree builder** runs in the browser with nothing to install. Build any tree, with payoffs, probabilities and costs on its branches, and it folds back and names the strategy.

ON THE SITE · the tree builder

Its Check tab hides the values and marks your own, node by node.

v348.jima.me/tree/

*Do the first fold-back by hand anyway. Software that prunes invisibly leaves you unable to say **why** a branch went, and Week 6 folds a revised tree on top of this one.*



In practice: where the numbers come from

IN PRACTICE

Probabilities are often subjective. “0.4 chance of a severe season” comes from history, expert judgment, or a forecast, not a known truth. Ask whose estimate it is and how it was made.

Payoffs hide assumptions. A \$120M benefit bundles prices, costs, and outcomes that can shift. Try plausible changes to them before trusting a recommendation.

Missing seasons. A history that dropped the worst years as “unusual” understates risk.

A tree hides nothing. Every assumption is a labelled branch, so a board can argue with a tree in a way it cannot argue with a cell formula.

Worth drawing even when the arithmetic is easy: every assumption ends up where someone can see it.

Back to the stockpile

- **Stockpile small, \$52M.** Three circles roll up to 30, 52, and 34; the square keeps the middle branch. Big held the only 120 and still lost: its -30 , weighted 0.6, takes 18 of the severe branch's 48 back. The picture says it as plainly as the table did.
- What the tree adds is not a different answer but **room**. The moment the county wants to decide something *after* the season shows its hand, the table has nowhere to put that choice and the tree simply grows another square.

Same numbers, same answer, a shape that can keep growing.

Wednesday

square

you decide: keep the
best branch

circle

nature decides: average
the branches

right to left

leaves first, first square
last

\$52M

stockpile small, table
and tree alike

*Wednesday puts a **square to the right of a circle**: decide, watch what the world does, decide again. That shape is where the value of waiting and the value of information both live.*

Group practice

IN CLASS

A student organization starting a weekend craft market can **sign** a full-season vendor contract now or **run one trial weekend**; turnout is Busy at 0.6 or Quiet at 0.4. Draw the tree from the table, fold it back by hand, **check your fold in the tree builder**, and explain your recommendation. The challenge: a busy trial weekend opens a **second square**; fold it, then build it in the builder. **Every member's name goes on the sheet**; one of you **uploads a photo of it to Canvas** before leaving. One participation point, half today's attendance record.

ON THE SITE · the tree builder

One laptop per group: steps 4 and 6 of the sheet happen here.

v348.jima.me/tree/



Photograph your attempt, then check it together

Two minutes to capture your work; eight for the instructor's demonstration

IN CLASS

Check every member's name and photograph your group's own attempt before the solution is shown. One member uploads the photo to Canvas by the end of class.

We close by checking the method, the main results with units, and a common mistake. Participation credit is for attempting the work.

Session 2

Wed Sep 23

Two ways not to commit blind

The question this session answers

PUZZLE

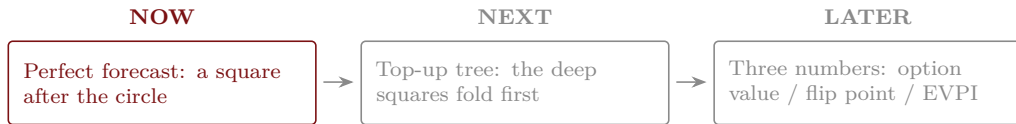
The county does not have to commit everything before the season shows its hand. It could **buy a seasonal forecast** before choosing. Or it could **buy small now and keep the right to top up** once the first weeks of the season are in.

Monday's tree had one square and gave one word: stockpile small, \$52M. Both of these plans put something *after* that square.

What is a forecast worth, what is the right to decide later worth, and how do you draw either one?

Where we are

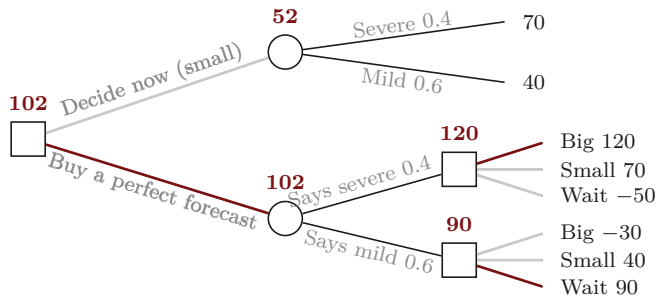
Three blocks today, in this order, all on the county



The first block compares deciding before and after learning the season to price a perfect forecast. “How do you draw the right to decide later” is the second, on the same county and the same numbers. The third prices that right, finds how far the 0.4 can move, and asks what the right does to the forecast’s worth.

A perfect forecast, drawn as a tree

Decide now, or hear the forecast and then decide; payoffs in \$M



1. Squares keep their best

says severe: best of 120, 70, -50 = **120**

says mild: best of -30, 40, 90 = **90**

2. Circles average

forecast: $0.4(120) + 0.6(90)$

$$= 48 + 54 = \mathbf{102}$$

decide now: $0.4(70) + 0.6(40)$

$$= 28 + 24 = \mathbf{52}$$

3. The first square keeps its best

best of 102 and 52 = **102**

Week 5 page, Session 2, Build the skill 1: every number on the tree, in the order the fold finds it.

Three doors, one \$50M

The tree gives the expected value of perfect information a third derivation

The table gap: EV with perfect information minus the best blind EV. $102 - 52 = 50$

The smallest EOL: regret is best payoff minus chosen payoff. Small loses $120 - 70 = 50$ in Severe and $90 - 40 = 50$ in Mild

The tree gap: “buy a perfect forecast” folds to 102 against 52 for “decide now.” Gap 50

*One idea, three doors: the profit a blind chooser leaves on the table. A real forecast is sometimes wrong, so it is worth **less** than 50; how much less is Week 6. The third block finds a second reason it can be worth less.*

Taylor 13e, pp.585 to 587. Notice the new shape: a square to the right of a circle is “deciding after you know,” and the rest of the session is about it.

The perfect forecast, on your laptop

IN CLASS

Open the perfect-forecast tree on the Week 5 page and press **Step one layer**, three times. The two lower squares pick big and wait *before* the circles average to 102 and 52; the readout prints each layer's arithmetic, and the table under the widget has every number. Then press **Reset** and **Fold back** to watch it run. That order, deepest squares first, is the whole method for the rest of today.

ON THE SITE · the perfect-forecast tree

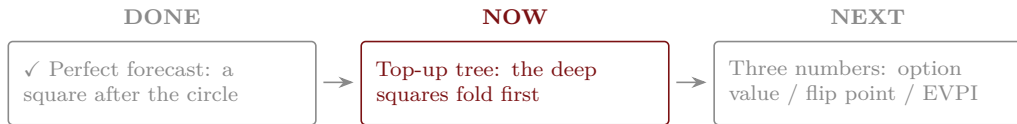
Session 2's first build, with the three doors written out under the widget.

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Where we are

The forecast is priced, two blocks to go



A perfect forecast folds to 102 against 52 for deciding now: a \$50M gain from choosing after learning the season. The county's other plan, buy small and top up once the season shows, is that shape with no forecast in it. Same county, same three options, same 0.4 and 0.6: one new price is all it needs.

Topping up mid-season

The one new price the county's second plan needs; payoffs in \$M

Buy small now, and once the first weeks of the season are in, the supplier will still sell, at an **emergency price**. Holding keeps Monday's numbers for stockpile small.

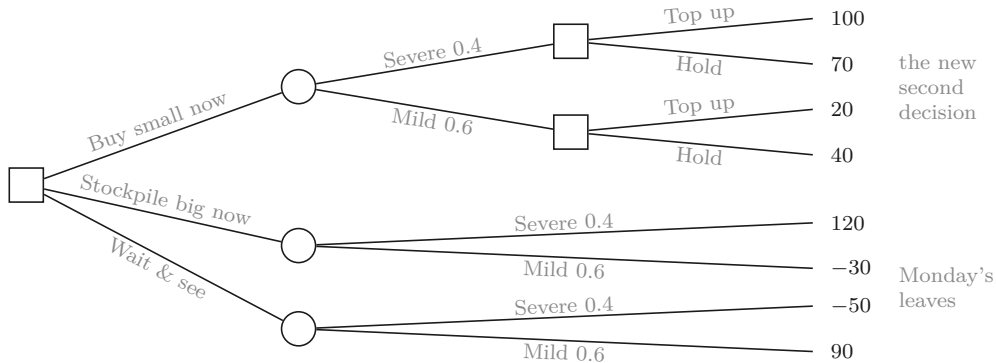
After the season shows	Top up	Hold
Severe (0.4)	100	70
Mild (0.6)	20	40

After a Severe read a top-up lands at 100, twenty short of the 120 that stockpiling big up front would have paid. After a Mild read it lands at 20, below holding's 40: it paid the emergency price for supply the season never needed.

*After buying small, each season offers a **choice** between two numbers, not one payoff. A payoff table has no way to say so; a tree does.*

Square, circle, square

The county's plan in the order things happen; payoffs in \$M

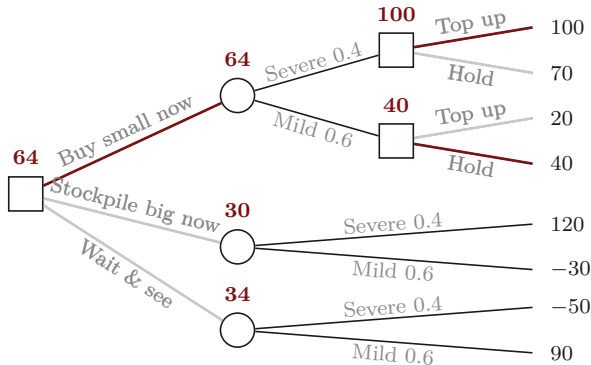


The second squares sit to the right of the season circle because the county chooses *after* the season shows.

Taylor 13e, pp. 593 to 594.

Fold it back, deepest squares first

The same two rules, applied at both square layers; payoffs in \$M



1. Deep squares keep their best

after Severe: best of 100, 70 = **100**

after Mild: best of 20, 40 = **40**

2. Circles average

buy small: $0.4(100) + 0.6(40)$

$$= 40 + 24 = \mathbf{64}$$

big: $0.4(120) + 0.6(-30) = \mathbf{30}$

wait: $0.4(-50) + 0.6(90) = \mathbf{34}$

3. The first square keeps its best

best of 64, 30 and 34 = **64**

Week 5 page, Session 2, Build the skill 2. The buy-small circle averages what the deep squares *chose*, never all four leaves.

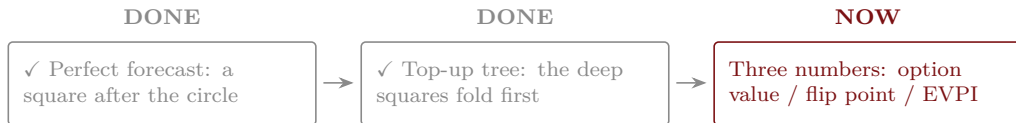
The answer is a strategy, not a word

- The surviving branch at **every** square, read left to right: *buy small now; if the season turns severe, top up; if it is mild, hold*. Worth \$64M, against \$52M for Monday's blind commitment.
- That is the **decision strategy**, a complete contingent plan. It names what to do now **and** what to do later for each outcome that could actually be reached.

“Buy small” alone is not an answer. A tree with a second square is answered by nothing less than the whole plan.

Where we are

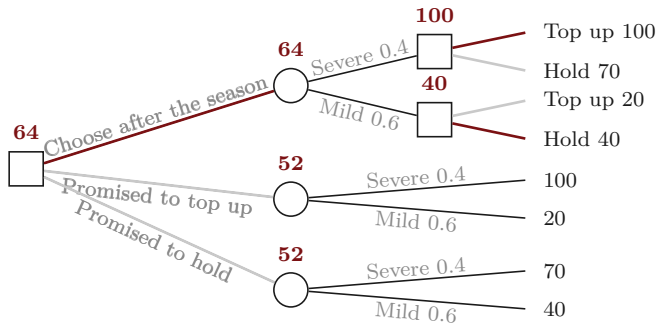
Two blocks down, one to go



The county's tree folds to 64 for buying small, against 34 for waiting and 30 for stockpiling big, and its answer is a sentence: buy small; if severe top up; if mild hold. The last block reads three more numbers off the same tree: the top-up right by itself, the odds at which the first move flips, and what knowing the season in advance would add now.

What the right to top up is worth

Three ways to buy small: keep the second decision open, or promise it away



Choose after the season

$$0.4(100) + 0.6(40) = 40 + 24 = \mathbf{64}$$

Promised to top up: Mild gets 20

$$0.4(100) + 0.6(20) = 40 + 12 = \mathbf{52}$$

Promised to hold: Severe gets 70

$$0.4(70) + 0.6(40) = 28 + 24 = \mathbf{52}$$

The square keeps the open plan

$$64 - 52 = \mathbf{12} \text{ from either promise}$$

\$12M is what the right to top up is worth. Not the 34 between buying small and big, which also carries two different supply plans.

Week 5 page, Build the skill 3; beyond the book, since Taylor (pp. 593 to 595) never prices the later decision alone.

From the tree to three lines

The county's top-up tree, deep squares already chosen: what each first move pays; in \$M

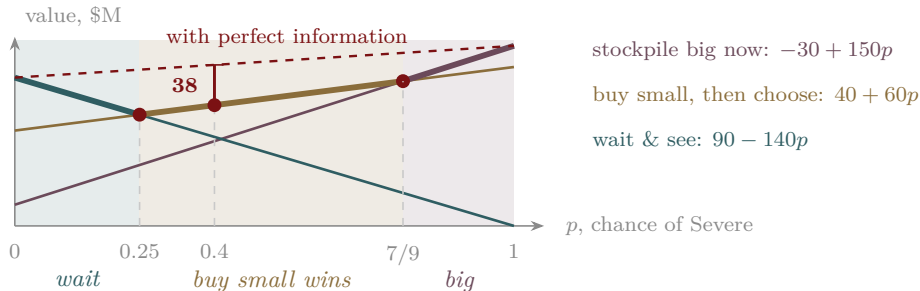
First move	Severe	Mild	As a line in p
Buy small, then choose	100 (top up)	40 (hold)	$100p + 40(1 - p) = 40 + 60p$
Stockpile big now	120	-30	$120p - 30(1 - p) = -30 + 150p$
Wait & see	-50	90	$-50p + 90(1 - p) = 90 - 140p$

Call the chance of a severe season p , so a mild one has $1 - p$, and weight each row by them. The deep squares' choices do not move with p : 100 beats 70 and 40 beats 20 whatever it is.

Every first move is a straight line in p , so the first move can change only where two lines cross.

How sure would you have to be to change the first move?

The county's top-up tree: each first move is a line in p , the chance of Severe

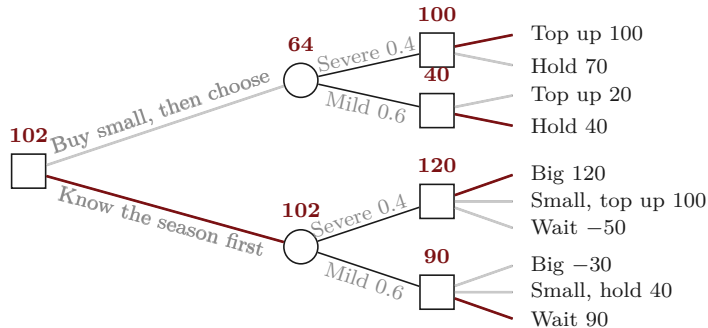


Wait meets buy small where $90 - 140p = 40 + 60p$: $p = 50/200 = \mathbf{0.25}$, both 55. Big meets it where $-30 + 150p = 40 + 60p$: $p = 70/90 = \mathbf{7/9}$, about 0.78, both about 86.7. The dashed line is perfect information; at 0.4 it sits 38 above buy small.

From 0.25 to 0.78, buy small and keep the top-up; the 0.4 sits inside. “Stockpile big” has to claim a severe season is likelier than seven in nine.

What would knowing the season be worth now?

The perfect-forecast shape again, with the top-up plan as the one to beat; payoffs in \$M



1. Squares keep their best
top-up plan: 100, then 40
Severe known: **120**, big
Mild known: **90**, wait

2. Circles average
 $0.4(100) + 0.6(40) = 64$
 $0.4(120) + 0.6(90) = 102$

3. The first square: 102
 $EVPI = 102 - 64 = 38$,
not $102 - 52$

Knowing the season, the county never needs the top-up. The right to top up has already bought 12 of the 50: $50 = 12 + 38$, so a forecast is now worth at most \$38M.

The county's tree, on your laptop

IN CLASS

Open the county's top-up tree on the Week 5 page and press **Step one layer**: the two deep squares choose first, top up after Severe and hold after Mild, then the circles average to 64, 30 and 34, and the first square keeps buy small. Fold the **three ways to buy small**: open, 64; promised to top up or to hold, 52. Drag the slider across **0.25** and **0.78** and the dot moves from wait to buy small to big. Fold **know the season first**: 102 against 64.

ON THE SITE · the county's top-up tree

The sequential tree, the three ways to buy small, the slider, and the know-first tree, each with its arithmetic written out beside it.

v348.jima.me/week/w05/



The same arithmetic in a benefits office

A digital application piloted in one district before a statewide launch

Plan, in \$k	Strong (0.6)	Weak (0.4)	Folded back
Launch statewide now	180	-120	60
Pilot, then expand or stop	expand 140	stop -20	76
Pilot, committed to expand	140	-160	20

The pilot costs \$20k and a year's delay costs \$20k more. Fold it the county's way: expand after strong (140 beats -20), stop after weak (-20 beats -160), so $0.6(140) + 0.4(-20) = 76$, against $0.6(180) + 0.4(-120) = 60$ for launching now. Committed to expand, $0.6(140) + 0.4(-160) = 20$.

The right to stop is worth $76 - 20 = \mathbf{56}$, more than the pilot's whole 16 lead: the county's square, circle, square, in a state office.

In practice: information is neither free nor perfect

IN PRACTICE

Imperfect signals. Real tests and forecasts are wrong sometimes. A model that ignores false positives and negatives overstates the value of information.

Sampling bias. A survey of people who answer the phone, or sensors only in wealthy neighborhoods, misreads the state of nature. Who is missing often matters more than who is in.

Do not over-test. Pay for information only when it could change what you do; once EVPI is small it cannot, so stop.

Expected value is the long-run rule, not always the right one. A county that one unprepared severe season could wipe out may rationally reject the EV-best plan. That is risk aversion: repeated, survivable decisions lean on expected value, one-shot ruinous ones on utility, which Week 6 measures.

The last point is the bridge to Week 6. Say it as a sentence, not as an apology for the model.

Back to the county

- The **right to top up later** is worth **\$12M**: buying small with that second decision open folds to \$64M, and either promise that closes it, always top up or never, drops the plan to Monday's \$52M.
- A **perfect** forecast is worth \$50M to a county that must commit blind, but only **\$38M** once it holds the right to top up. A real forecast is worth less still; Week 6 makes that precise with EVSI.
- So the county **buys small now, tops up if the season turns severe, and holds if it does not**; the plan stands while p stays between 0.25 and 0.78, and it declines any forecast priced above \$38M.

Decision analysis turned “how much is it worth to know more, or to keep our options open?” into dollar figures on one tree.

This week's take-home

Two files, on Canvas, by Sunday at 11:59 PM

- A hospital system: **pilot a telehealth service** in one division and decide again once uptake is known, or roll out now. Name the squares and the circles, build the probability from a peer record, net the payoffs, name the unused figure.
- Fold the tree with **live formulas** and check it in the **tree builder**, whose **picture** of your folded tree is the second file. Then today's three numbers: option value, flip probability, and what knowing uptake in advance is worth.
- Block D wants the **strategy**: now, after a High read, after a Low read. "Pilot" alone is not an answer there either.

ON THE SITE · the brief and the workbook

Download the workbook here. **Tier 1**: AI as a tool, disclosed.

v348.jima.me/week/w05/



Carry into Week 6

a strategy

now, then if Severe, then
if Mild

\$12M

the right to top up,
priced alone

0.25 to 0.78

where buying small stays
best

\$38M

what knowing is worth
now, not 50

*Next week the forecast stops being perfect. An **imperfect signal** revises the probabilities on the circles, and the tree prices it: the expected value of sample information, always below EVPI.*

Review

Week 5

Can you define these without looking?

Ten terms this week introduced

decision tree	decision node (square)	probability node (circle)	branch and leaf
fold back	decision strategy	sequential decision	value of the second decision
flip probability	EVPI on a tree		

ON THE SITE · Key Concepts

Most of these have a one-line definition and a worked example behind them (filter by week); the two frames after this one define them all.

v348.jima.me/concepts/



Definitions: the drawing

Decision tree	Choices, uncertain events, probabilities, and payoffs drawn left to right in the order they happen
Decision node	A square: a fork you control. Folded back by keeping the best branch
Probability node	A circle, also called a chance node: a fork nature controls, with a probability on every branch out of it, summing to 1. Folded back by averaging
Branch, leaf	One alternative out of a square or one state out of a circle; a leaf is a branch that ends, carrying its payoff
Fold back	Solve from the right: average at every circle, keep the best branch at every square, until the first square is reached

Definitions: what the tree can say that a table cannot

Sequential decision	A problem where an early choice or observation changes the options at a later decision point: square, circle, square
Decision strategy	The surviving branch at every square, read left to right: what to do now and after each outcome that could be reached
Value of the second decision	Fold the tree with the later choice open, then with it forced, and take the gap. Change one thing only
Flip probability	The probability at which the two first-stage arms are worth the same, so the first decision changes
EVPI on a tree	Re-decide the first square under each outcome, average those best plans by the odds, and subtract the best plan without information

Test yourself

If one of these is slow, that is where to reread

1. A colleague averages the two branches out of the county's Severe square, top up and hold. What number do they get, and what does that price the county as doing?
2. Why must the county's top-up squares sit to the right of the season circle rather than to its left?
3. "Buy small" is the county's first move. Write the whole strategy, and say why the shorter answer is not an answer.
4. Buying small beats stockpiling big by \$34M and the right to top up is worth \$12M. Why are those different numbers, and which one changed only one thing?
5. A perfect forecast is worth \$50M against Monday's blind plan but only \$38M against the top-up plan. Why did it fall, and where did the other \$12M go?

Numbers worth remembering

52 to 64

the county, once it may
top up later

102 vs 52

perfect forecast vs
decide now: EVPI 50

\$12M

the right to top up,
priced alone

0.25 to 0.78

where buying small
stays best

Question

The number

The move

The right to top up

\$12M

open 64 minus promised 52, either
promise

Where the first move flips

0.25, 7/9

$40 + 60p$ set equal to each rival's line

Knowing the season, now

\$38M

$0.4(120) + 0.6(90) = 102$, minus 64

Knowing it, committing blind

\$50M

the same 102, minus Monday's 52

Carry into Week 6

- Week 6 makes the forecast **imperfect**. A signal that is sometimes wrong revises the probabilities on the circles, prior to posterior, by Bayes' rule.
- The revised tree is folded back exactly the way this week's were, and the gap it opens is the **expected value of sample information**, always below EVPI.
- Then utility: why a county that could be ruined by one bad season may rationally refuse the plan with the highest expected value.

ON THE SITE · Week 6

The reading and the before-class self-check are posted. Do the self-check before Monday.

v348.jima.me/week/w06/

