
SPEA-V348 Management Science · Week 7 review

Linear programs, and the corner that wins

Both sessions, every slide, fully revealed. Read it without me in the room.

Mon Oct 5 and Wed Oct 7, 2026

The week at a glance

Mon Oct 5	Three ingredients in order: decision variables, objective, constraints. Two lines the brief left out, as two more rows. The four assumptions, and what is legal
Wed Oct 7	The feasible region, the profit line, the corner-point method, slack, the three irregular cases, then the catering order: minimization, surplus, whole batches
Reading	Taylor 13e, Chapter 2, pp. 54 to 80, straight through
Due	The Week 7 workbook, on Canvas. Tier 1: AI as a tool, with disclosure

ON THE SITE · the Week 7 page

The three widgets, the two self-checks, and the workbook download all live here.

v348.jima.me/week/w07/

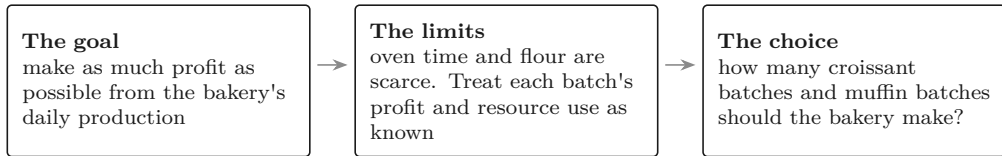


Session 1

Mon Oct 5

Choosing a production plan

Known inputs, scarce resources, and quantities you control



*Today you learn to **write the model down**. Wednesday you solve it on a graph.*

The corner bakery

The question this session answers

PUZZLE

Each morning you bake **croissant batches** and **muffin batches**. A croissant batch earns **\$4** profit, a muffin batch **\$3**. The oven gives you **10 hours** (2 per croissant batch, 1 per muffin batch) and you have **15 units of flour** (1 per croissant batch, 3 per muffin batch).

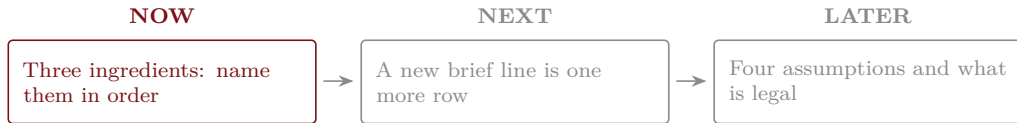
How many batches of each should you bake to make the most profit?

A toy stand-in for a serious idea. Production plans, feed blends, refinery mixes, and portfolios are all, structurally, this question: use scarce resources to do the most good.

*You cannot reliably eyeball it. By Wednesday you will have solved it **exactly**.*

Where we are

Three blocks today, in this order



The bakery's question, how many batches of each, becomes four lines of symbols in the first block. The second block hands you two lines the brief left out and shows the four lines grow without a new idea, and the third says when the model is telling the truth.

Try one plan before writing any variables

2 croissant + 3 muffin batches	Calculation	Result	Check
Profit	$4(2) + 3(3)$	\$17	what it earns
Oven	$2(2) + 1(3)$	7 hours	$7 \leq 10$
Flour	$1(2) + 3(3)$	11 units	$11 \leq 15$

The plan fits. Replacing 2 and 3 with x and y lets the same three calculations test any plan.

Week 7 page, the three ingredients.

The three ingredients of every linear program

Naming them, in this order, is the skill

1. Decision variables,

first, always

what you control: how much to make, buy, ship, or hire.

Here x croissant batches, y muffin batches

2. Objective

what you want most or least of, as a weighted sum of the variables: maximize $Z = 4x + 3y$, profit in dollars

3. Constraints

the limits, as linear inequalities: $2x + y \leq 10$ for the oven, $x + 3y \leq 15$ for the flour, and $x, y \geq 0$

*The order matters. You cannot write an objective or a constraint until you have decided **what the variables are**.*

Taylor 13e, pp. 54 to 58. The book's Beaver Creek Pottery is the same shape: two products, two scarce resources.

A constraint, in symbols

Where every number in the brief lands

$$2x + 1y \leq 10$$

The coefficients: every *per-unit* number. 2 oven-hours per croissant batch, 1 per muffin batch

The right-hand side: the *available total*. 10 oven-hours a morning

The direction: $\leq$ says “use no more than”. A requirement you must meet would say $\geq$

The left side is what the plan *uses*; the right side is what you *have*. Taylor 13e, pp. 55 to 57. Non-negativity, $x, y \geq 0$, says you cannot bake a negative batch.

The bakery, written out

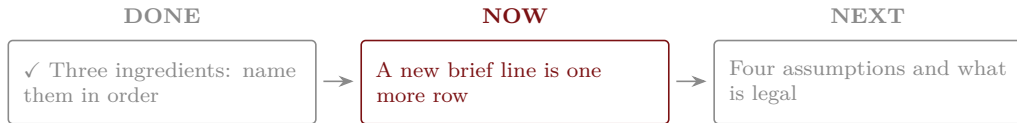
Four lines, and you have done the hard part

Ingredient	In symbols	From the brief
Decision variables	x croissant batches, y muffin batches	what you choose each morning
Objective	maximize $Z = 4x + 3y$	\$4 and \$3 profit per batch
Oven	$2x + y \leq 10$	2 h and 1 h per batch, 10 h available
Flour	$x + 3y \leq 15$	1 and 3 units per batch, 15 on hand
Non-negativity	$x \geq 0, y \geq 0$	no negative batches

*You do not know x and y yet. **The model now states the decision precisely**, and it is done.*

Where we are

One block down, two to go



The bakery is four lines: x and y in batches, maximize $Z = 4x + 3y$, and two limits with every per-unit number on the left and every total on the right. Whether you can do it again on two lines the brief left out is what the next block tests.

The brief grows by two lines

Same bakery, same moves: write them before they appear

Ingredient	In symbols	From the brief
Eggs	$x + 2y \leq 12$	1 and 2 eggs per batch, 12 in the fridge: a limit, so $\leq$
Café order	$y \geq 2$	at least 2 muffin batches: a requirement, so $\geq$

*Per-unit numbers on the left, the total on the right, and the one **requirement** turned the sign around. Two more rows, and no new idea.*

The expanded bakery model

Ingredient	In symbols	From the brief
Decision variables	x croissant batches, y muffin batches	unchanged
Objective	maximize $Z = 4x + 3y$	unchanged
Oven	$2x + y \leq 10$	unchanged
Flour	$x + 3y \leq 15$	unchanged
Eggs	$x + 2y \leq 12$	new: a limit, so $\leq$
Café order	$y \geq 2$	new: a requirement, so $\geq$
Non-negativity	$x \geq 0, y \geq 0$	unchanged

Two rows joined the table; nothing above them moved.

Where we are

Two blocks down, one to go



Two lines the brief left out became two rows: eggs $x + 2y \leq 12$, because a fridge is a limit, and the café's $y \geq 2$, because a standing order is a floor. What those six lines take on faith about the world, and which lines are not allowed at all, is what the last block says.

What linear programming assumes

Four things that have to be true of the world for the model to be honest

Proportionality double the batch, double its profit and its resource use. No volume discounts, no economies of scale	Additivity total profit is the sum of each product's; products do not interact	Divisibility fractions are allowed: 3.5 batches is a legal answer. Whole units are Week 10	Certainty the coefficients and the right-hand sides are known, fixed numbers
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Proportionality is why profit is $4x$ and not $4x^2$. Break an assumption and the model still returns a confident number; it is just the wrong one.

Taylor 13e, pp. 79 to 80.

What counts as a legal linear program

One rule follows from all four assumptions

Expression	Legal?	Why
$5x + 3y + \frac{1}{3}z$	yes	a plain weighted sum of the variables
$5xy$	no	a product of two variables
$5x/y$	no	a ratio of variables
x^2	no	a variable squared

*An objective or a constraint is a **plain weighted sum**. Multiply or divide one variable by another and you have left linear programming behind.*

The same arithmetic in a crisis team

The finished template, handed to a public agency

Ingredient	In symbols	From the brief
Decision variables	x daytime outreach shifts, y evening crisis-team shifts	the two kinds of shift to staff
Objective	maximize $6x + 9y$	6 and 9 impact points a shift
Staff-hours	$2x + 3y \leq 36$	2 and 3 hours a shift, 36 available
Dispatches	$x + y \leq 14$	14 vehicles can go out

*The algebra is the bakery's. The **judgment** is whether “impact points” and the two limits are credible numbers.*

This one ties: every plan from (0, 12) to (6, 8) scores 108, because the objective is three times the staff-hours line. Wednesday names that case.

In practice: formulation is where the real errors hide

IN PRACTICE

Where do the coefficients come from? Profit per batch depends on costs and sale price. Fixed rent does not change the winning plan; subtract it to report net profit. Convert weekly limits before using them in a daily model.

A missing constraint is the most dangerous error. Forget the flour limit and the model happily recommends an impossible plan. Listing every scarce resource is the formulator's real job.

The model is not the world. It optimizes what you wrote down. If freshness, staff morale, or a contract minimum matters and you left it out, the “optimal” plan can still be a bad decision.

Sanity-check the answer against the things the model was not told.

Back to the bakery

- You can now state the problem precisely, and you know it is a **legal** linear program: a plain weighted sum, maximized over linear limits.
- You do not have the numbers x and y . You have something more reusable: a **template** that took two more brief lines, and then a crisis team's shifts, without a new idea. Wednesday finds the corner on the original four lines.

ON THE SITE · the Week 7 page

The bakery model, its two extra brief lines, the four assumptions, and the legal-or-not table, all written out.

v348.jima.me/week/w07/



Wednesday

variables

named first, with units

coefficient

every per-unit number

RHS

the right-hand side:
every available total

$\leq$ or $\geq$

a limit, or a requirement

*Wednesday draws the two constraints as lines, shades the plans that respect both, and slides the profit line until it leaves the region. **The last point it touches is the answer.***

Group practice

IN CLASS

The student-union lunch counter sells **veggie wraps** and **rice bowls** at \$5 and \$7 profit. Prep time and the warming tray run short: a wrap takes 2 minutes and 1 tray slot, a bowl 3 minutes and 2 slots, with 60 minutes and 40 slots over the rush. **Do not solve it.** Write the three ingredients, with units on the variables and each constraint's per-unit numbers on the left and its total on the right, and name the assumption that lets 3.5 wraps be an answer. **Every member's name goes on the sheet**; one of you photographs it and **uploads the photo to Canvas** before leaving. One participation point, half today's attendance record.

ON THE SITE · Week 7 practice sheet

The same sheet is on the site if you want it on a screen.

v348.jima.me/assets/docx/v348-w07-s1-group-practice.docx



Photograph your attempt, then check it together

Two minutes to capture your work; eight for the instructor's demonstration

IN CLASS

Check every member's name and photograph your group's own attempt before the solution is shown. One member uploads the photo to Canvas by the end of class.

We close by checking the method, the main results with units, and a common mistake. Participation credit is for attempting the work.

Session 2

Wed Oct 7

Infinitely many plans

The question this session answers

PUZZLE

Same bakery, same model: maximize $4x + 3y$ with $2x + y \leq 10$ and $x + 3y \leq 15$. There are infinitely many plans that respect both limits: bake 1 and 1, or 2 and 3, or 0.5 and 4.8, or ...

Guessing finds a plan. It does not find the best one, and it never tells you that there is no better plan you did not try.

*Which plan is best, and how can you be **sure** there is not a better one?*

Where we are

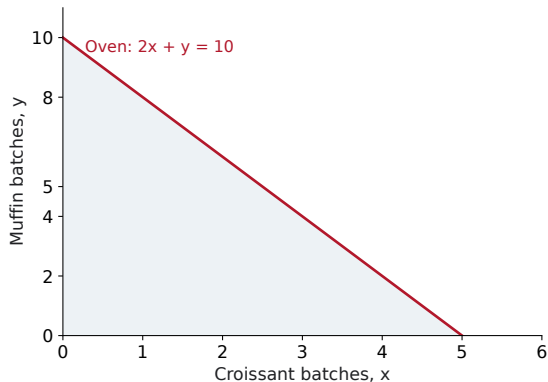
Three blocks today, in this order



The puzzle asks which plan is best and how you can be sure. The first block answers both with one picture: the best plan is a corner, and there are only four to check. The second reads what that corner uses up and what can go wrong, and the third turns the picture around for the bakery's catering order, a cost you push inward.

Build the region: the oven boundary

Find its intercepts, then test which side is allowed



1. One resource limit

Oven intercepts:

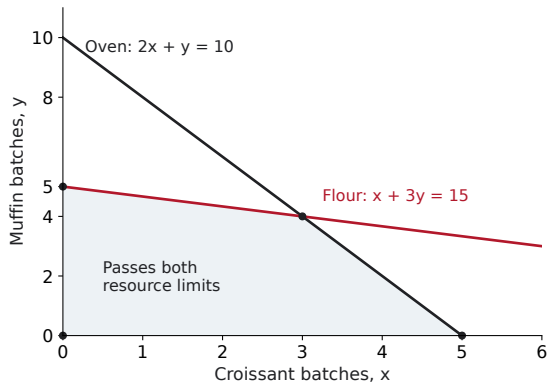
$(5, 0)$ and $(0, 10)$.

Test $(0, 0)$: $0 \leq 10$.

Keep the shaded side.

Build the region: add flour

A feasible plan must satisfy both resource limits



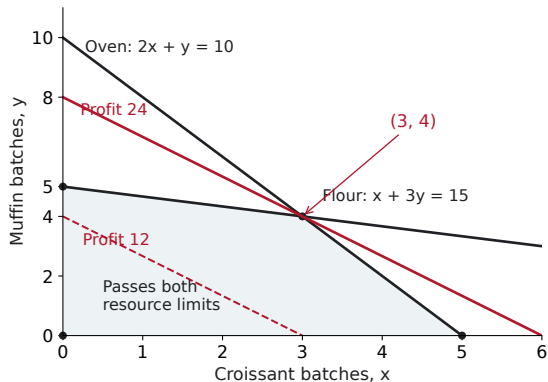
2. Keep the overlap

Add the flour boundary.
Its x-intercept, (15, 0),
is outside this view.

(0, 10) fails flour:
 $0 + 3(10) = 30 > 15$.

With the region built, raise profit

Keep the limits fixed and move only the objective line



3. Raise profit

Resource limits stay put.
Only the profit line moves.

At (3, 4):

$$4(3) + 3(4) = 24.$$

No feasible plan earns more.

The sliding line, on your laptop

IN CLASS

Open the first widget on the Week 7 page. Drag the profit slider and watch the orange objective line move across the region with its slope fixed. Then press **Maximize** and watch it stop: the last corner it touches is $x = 3$, $y = 4$, and the readout says $Z = 24$. The slider will not go past 24, because no plan in the region does.

ON THE SITE · the graphical widget

The bakery region with the profit line live, and the corner table written out below it.

v348.jima.me/week/w07/



The corner-point method, when there is no slider

On paper, or in an exam

- **A.** Plot the constraints. **B.** List the corners of the feasible region.
- **C.** Read x and y at each corner. **D.** Compute the objective Z at each.
- **E.** Pick the best. **F.** State the decision in words, with units.

*For this bounded feasible region, a corner attains the optimum, checking the handful of corners is a **proof**, not a guess.*

Taylor 13e, pp. 62 to 66. A corner is where two boundary lines meet, an axis counting as a line.

Every corner of the bakery

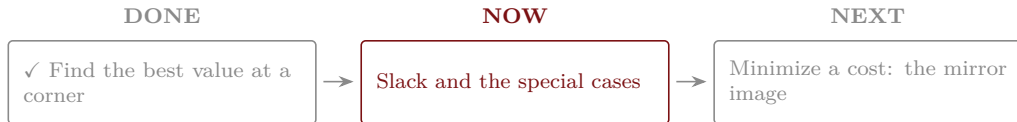
Corner (x, y)	How it arises	$Z = 4x + 3y$
$(0, 0)$	both axes: make nothing	0
$(5, 0)$	oven line meets $y = 0$	20
$(3, 4)$	oven and flour lines meet	24
$(0, 5)$	flour line meets $x = 0$	15

The corner where the two limits cross is found by solving them together: $2x + y = 10$ and $x + 3y = 15$ give $x = 3$, $y = 4$. The oven line's own axis point $(0, 10)$ is not a corner: it would use 30 units of flour against 15.

*Bake **3 croissant batches** and **4 muffin batches** for **\$24**. The widget and the table agree, and the table needs nothing but arithmetic.*

Where we are

One block down, two to go



Bake 3 croissant batches and 4 muffin batches for \$24, found twice: the sliding line stopped at (3,4), and the corner table put 24 above 20, 15, and 0. What that corner uses up, and what a worse corner leaves idle, is a question the table did not answer.

Slack, in symbols

Which resources are you actually using up?

$$\text{slack} = \text{RHS} - \text{used}$$

What you had: the right-hand side, the oven's 10 hours

What you used: the left side at the plan. At $(3, 4)$, $2(3) + 4 = 10$ oven-hours

The leftover: $10 - 10 = 0$. Zero slack means the constraint is *binding*

Taylor 13e, pp. 67 to 69. A slack variable turns $2x + y \leq 10$ into the equation $2x + y + s = 10$, with $s \geq 0$ recording the unused hours.

Binding at the best corner, slack at a worse one

Constraint	At the optimum (3,4)	At the corner (5,0)
Oven, $2x + y \leq 10$	$10 - 10 = 0$: binding	$10 - 10 = 0$: binding
Flour, $x + 3y \leq 15$	$15 - 15 = 0$: binding	$15 - 5 = 10$: ten units idle

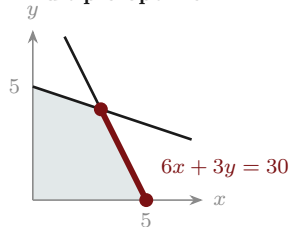
At (3,4) every oven-hour and every unit of flour is spent. At (5,0) ten units of flour sit unused, while the plan earns less; slack alone does not prove inferiority.

Here both resources bind at the optimum. Extra capacity on a constraint with slack does not improve the best value; check what is binding before paying for more.

Three ways a linear program can surprise you

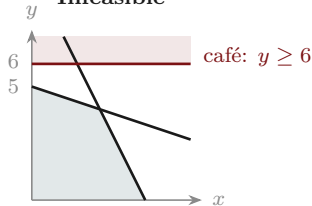
Each one is the bakery with a small change to its model

Multiple optima



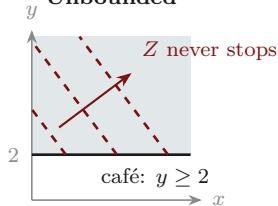
at \$6 a croissant batch, the whole oven edge ties at \$30

Infeasible



the café wants $y \geq 6$, but flour caps muffins at $15/3 = 5$

Unbounded



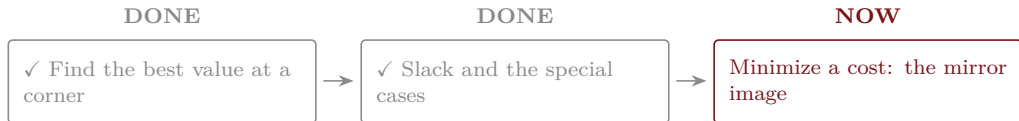
the café's $y \geq 2$ as the only line: no oven, no flour

A tie gives several best plans. Infeasible requirements conflict; unbounded profit prompts a missing-limit check.

Taylor 13e, pp. 76 to 79. All three pictures use the bakery's variables.

Where we are

Two blocks down, one to go



At $(3, 4)$ both limits bind with zero slack, and each special case was the bakery with a small change to its model. Every picture so far pushed a profit line outward against ceilings. The bakery's evening catering order brings floors instead, minimums to meet at the lowest cost, so the last block turns the picture around and pushes a cost line inward.

The other half of the chapter: minimize a cost

The bakery's catering order

PUZZLE

An office park has a standing weekly order with the bakery: **at least 16 breakfast trays** and **at least 12 lunch boxes**, for a fixed fee. A croissant batch fills 3 trays and 1 box and costs \$5 to bake; a muffin batch fills 1 tray and 3 boxes and costs \$3. It is baked in the evening, so the morning's oven and flour play no part. What is the cheapest way to fill it?

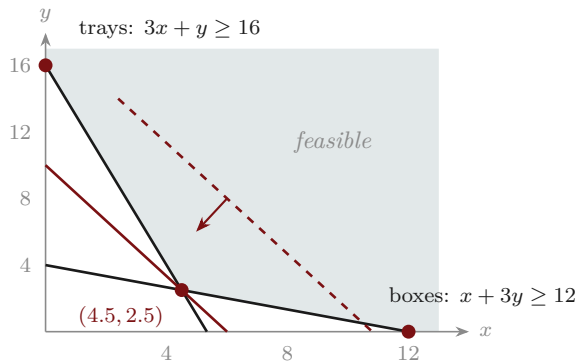
$$\text{minimize } 5x + 3y \quad \text{subject to} \quad 3x + y \geq 16, \quad x + 3y \geq 12, \quad x, y \geq 0$$

- Same x and y , same three ingredients. The fee is fixed, so every dollar saved on baking is profit: the objective is a cost to **minimize**. The order sets floors, so the tray line and the box line are both $\geq$.

Taylor 13e, pp. 70 to 75. The same bakery, asked a cost question instead of a profit one.

The catering order, drawn

Floors instead of ceilings, and a cost pushed inward



Floors, not ceilings. $A \geq$ keeps the far side, so the region sits above both lines

No last point outward. A line pushed out never leaves, so push the cost line *in*

The last point it touches going in is the cheapest corner: $(4.5, 2.5)$, \$30

The morning run's picture, turned around: floors instead of ceilings, and a cost line pushed in instead of a profit line pushed out.

Taylor 13e, pp. 70 to 75, redrawn from the catering order's own numbers.

Every corner of the catering order

Check them all: the crossing is not automatically the answer

Corner (x, y)	How it arises	Cost $5x + 3y$
$(0, 16)$	tray line meets $x = 0$	\$48
$(4.5, 2.5)$	tray and box lines meet	\$30
$(12, 0)$	box line meets $y = 0$	\$60

The crossing solves the two floors together: the tray line gives $y = 16 - 3x$, so $x + 3(16 - 3x) = 12$, $x = 4.5$, $y = 2.5$. Here it wins, 4.5 croissant batches and 2.5 muffin batches for \$30. On other problems the minimum sits at an axis corner, making none of one product, which is exactly why you check every corner rather than assume.

*Your take-home's shelter blend is one of the other problems: its crossing is **not** the cheapest corner.*

Surplus, the mirror of slack

- A $\geq$ requirement becomes an equation by **subtracting** the amount above the minimum: $3x + y - s = 16$ with $s \geq 0$. The **surplus** s records trays beyond the 16 ordered.
- At (4.5, 2.5) both floors are met exactly, so both surpluses are 0. At (12, 0) the trays come to 36 against 16, a surplus of 20 trays you paid to bake and nobody ordered.

Slack is what you had and did not use. Surplus is what you delivered and were not asked for. Solver reports both.

The cost line, on your laptop

IN CLASS

Open the minimization widget on the Week 7 page. The cost line opens at \$60, through (12, 0). Drag the slider down or press **Minimize** and watch it slide *toward* the origin, the mirror image of the morning run, and stop at $x = 4.5$, $y = 2.5$ with the readout at $Z = 30$.

ON THE SITE · the minimization widget

The catering order's region, with the three corners costed out just below it.

v348.jima.me/week/w07/



What if you cannot bake half a batch?

- Rounding **down** to $(4, 2)$ bakes only $3(4) + 2 = 14$ trays and $4 + 3(2) = 10$ boxes, short of both floors. It does not fill the order at all.
- Rounding **up** to $(5, 3)$ fills it and costs \$34.
- The cheapest whole-batch plan is neither: $(4, 4)$ at **\$32**, 16 trays and 16 boxes, sitting on the tray line exactly. It is not a rounding of the fractional answer.

*Rounding is not reliable. Week 10 puts the whole-number requirement **inside** the model instead of patching it afterwards.*

Whole batches, on your laptop

IN CLASS

Open the third widget on the Week 7 page: every dot is a whole-batch plan that fills both floors. Press **LP optimum**, then **Round it** and read that $(5, 3)$ stays legal but overpays by \$2, then **Integer optimum** and watch it land on $(4, 4)$, a different plan from any rounding. Each press draws the cost line through its plan: at \$32 no dot sits on the cheaper side.

ON THE SITE · the whole-number widget

The lattice of whole-batch plans, with the three candidates worked out under it.

v348.jima.me/week/w07/



Back to the bakery

- **3 croissant batches and 4 muffin batches, \$24**, found two ways that agree: the sliding profit line, and the corner table you can do by hand. You can be sure, because a corner attains the best value here and the table checked all four.
- Both limits bind. There is no slack to spare; the plan squeezes every oven-hour and every unit of flour.
- The same picture, turned around, fills the catering order: $(4.5, 2.5)$ for \$30, or $(4, 4)$ for \$32 if the batches must be whole.

*Real problems have dozens of variables, one axis each, too many to graph. Next week the **same three ingredients** go to Solver, which walks the corners for you and reports every slack.*

This week's take-home

One file, on Canvas, by Sunday at 11:59 PM

- Two briefs: a **campus recycling center** to maximize and an **animal shelter's feed blend** to minimize. Neither model is written for you; one capacity is quoted per day for a weekly plan. No Solver, anywhere.
- Graph each region. On the recycling center, **enumerate every corner yourself**: nothing is named. One of its three limits turns out not to matter, and you say which and by how much.
- On the blend, cost the three corners the workbook lists: the crossing is **not** the answer, and the cheapest mix over-delivers one minimum, so report that surplus.

ON THE SITE · the brief and the workbook

Download the workbook here. **Tier 1**: AI as a tool, disclosed.

v348.jima.me/week/w07/



Carry into Week 8

a corner

attains the best value
here

slack

had minus used; surplus
is its mirror

$\geq$ flips it

region above the lines,
slide inward

(4, 4)

whole batches: no
rounding of (4.5, 2.5)

*Next week Excel's **Solver** takes the three ingredients as three kinds of cell, walks the corners, and prints an Answer Report that names which limits bind.*

Review

Week 7

Can you define these without looking?

linear program	decision variable	objective function	constraint
right-hand side	non-negativity	feasible region	corner point
slack	surplus	binding constraint	the four assumptions
multiple optima	infeasible problem	unbounded problem	corner-point method

ON THE SITE · Key Concepts

Most of these have a one-line definition and a worked example behind them; the two frames after this one define them all.

v348.jima.me/concepts/



Definitions: writing the model

Linear program	Choose decision variables to maximize or minimize a linear objective, subject to linear constraints
Decision variable	What you control, named first and with units: x croissant batches a morning
Objective function	The weighted sum you push up or down: $Z = 4x + 3y$, with a per-unit number on every variable
Constraint	A limit or a requirement: per-unit numbers left, the available total (the right-hand side) right, plus non-negativity
The four assumptions	Proportionality, additivity, divisibility, certainty. Together: a plain weighted sum of known numbers, fractions allowed
Standard form	Every variable on the left, the constant on the right; with a slack (or surplus) variable, each inequality becomes an equation

Definitions: solving it on a graph

Feasible region	Every plan that respects every constraint at once. Each $\leq$ keeps the side toward the origin
Corner point	Where two boundary lines meet, an axis counting as a line. The optimum is one, so the corner-point method costs them all
Slack	What you had minus what you used, on a $\leq$ constraint. Zero slack means the constraint is binding
Surplus	What you delivered minus what was required, on a $\geq$ constraint
Multiple optima	The objective line is parallel to a binding constraint, so a whole edge ties. A real answer
Infeasible, unbounded	An empty region, or an objective that rises forever. Both mean the model was written wrong

Test yourself

If one of these is slow, that is where to reread

1. A brief quotes a capacity per day and asks for a weekly plan. Which ingredient does that touch, and what goes wrong if you skip the conversion?
2. Why is $5xy$ not allowed in an objective, and which of the four assumptions does it break?
3. The profit line leaves the bakery region last at $(3, 4)$. Why can you be sure no interior point does better?
4. At the corner $(5, 0)$ the flour slack is 10. Say in words what those 10 units are, and why the plan is inferior.
5. On the catering order the crossing wins at \$30. Why must you still cost $(0, 16)$ and $(12, 0)$ before saying so?

Numbers worth remembering

$(3, 4)$, \$24

the bakery, both limits
binding

$(4.5, 2.5)$, \$30

the catering order, at
the crossing

$(4, 4)$, \$32

whole batches, no
rounding of 4.5

10 units

flour idle at the corner
 $(5, 0)$

Question

Bakery

The move

Which plan is best

$(3, 4)$

cost every corner: 0, 20, 24, 15

Where do the two lines cross

$(3, 4)$

solve $2x + y = 10$ and $x + 3y = 15$
together

Which resources are used up

both

slack 0 on the oven and on the flour

Carry into Week 8

- Week 8 hands the **same three ingredients** to Excel's Solver: changing cells for the variables, an objective cell, and one left-hand-side cell per constraint.
- Solver walks the corners for you and prints an **Answer Report** naming which constraints bind and how much slack the others carry.
- Standard form matters there too: every variable on the left and the constant on the right, so each constraint is one formula cell against one number. You add no slack variables; Solver reports the slack itself.

ON THE SITE · Week 8

The reading is eight pages, half of them skippable. Do the self-check before Monday.

v348.jima.me/week/w08/

