
SPEA-V348 Management Science · Week 9 review

What the Sensitivity Report says

Both sessions, every slide, fully revealed. Read it without me in the room.

Mon Oct 19 and Wed Oct 21, 2026

The week at a glance

Mon Oct 19	Coefficient ranges, shadow prices, resource ranges, reduced cost and the Sensitivity Report; the garden practice and closing check
Wed Oct 21	TA-administered midterm only: setup from 2:20 PM, planned testing 2:40 to 3:10 PM, submission checks through 3:35 PM; Weeks 1 to 8
Reading	Taylor 13e, Chapter 3, pp. 104 to 115, and the product-mix and diet examples in Chapter 4, pp. 135 to 143
Due	The Week 9 workbook, on Canvas. Tier 1: AI as a tool, with disclosure

ON THE SITE · the Week 9 page

The two widgets, the two self-checks, the workbook download, and the midterm note all live here.

v348.jima.me/week/w09/

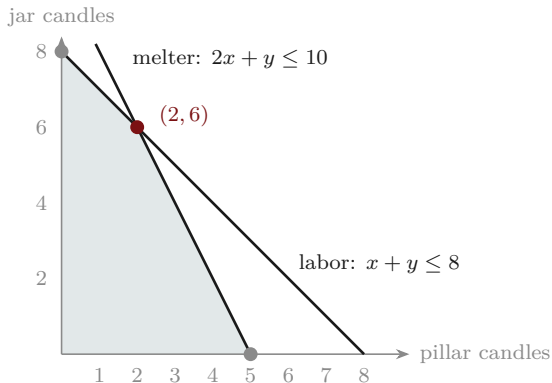


Session 1

Mon Oct 19

One candle workshop, all week

Pillar and jar candles, a wax melter, a labor pool and fragrance oil, drawn as a region



The model: maximize $4x + 3y$
for x pillar and y jar candles, with
melter $2x + y \leq 10$, labor $x + y \leq 8$
and fragrance oil $x + 3y \leq 40$

The corners: (5, 0) earns 20, (0, 8)
earns 24, and the crossing (2, 6) earns
26

Solver's plan: 2 pillar and 6 jar
candles, \$26. Melter and labor bind;
fragrance oil uses 20 of 40 ounces

*Hold that plan; Monday asks how input changes affect it. The report reads right only in **standard form**: variables left, the constant alone on the right.*

The price of wax is on the move

The question this session answers

PUZZLE

Your pillar candles are poured in beeswax, whose price the supplier resets every month, and your accountant says the profit on a pillar candle might drift anywhere from about **\$3 to \$5** this month.

Does that mean re-planning the workshop every time the number moves? The plan, 2 pillar candles and 6 jar candles, was computed at exactly \$4.

*At what point does the **optimal plan itself** change, and not just the profit total?*

Where we are

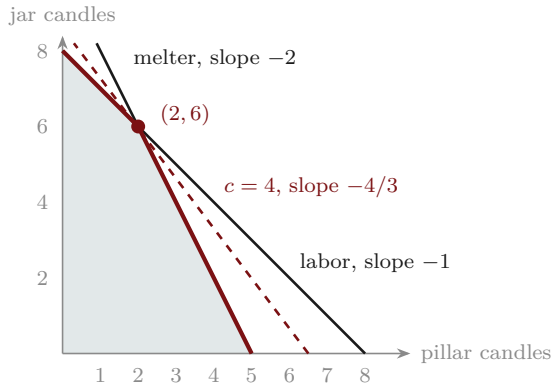
Three questions on one workshop



Start with the candle workshop at (2,6) for \$26. First move a profit rate, then a resource total, then ask why a tea-light pack stays out.

The profit line tilts; the corner holds, until it does not

The same region, with the objective at three pillar-candle profits



At \$4 the slope $-4/3$ sits between the melter's -2 and labor's -1 , so $(2, 6)$ is the last point touched

Raise it to \$6 and the objective lies along the melter edge: $(2, 6)$ ties $(5, 0)$. Past \$6 the plan jumps there

Lower it to \$3 and it lies along the labor edge: $(2, 6)$ ties $(0, 8)$. Below \$3 the plan jumps there

Taylor 13e, pp. 104 to 108. The corner is fixed by the lines that meet there, not by the exact profits.

The allowable range, in symbols

The band of pillar-candle profits over which 2 pillar and 6 jar candles stays optimal

$$\text{\$3} \leq c_{\text{pillar}} \leq \text{\$6}$$

Beat all jars, (0, 8): $2c + 18 \geq 24$,
so $c \geq 3$. At \$3 both plans earn \$24.
Allowable decrease from \$4: 1

Beat all pillars, (5, 0): $2c + 18 \geq 5c$,
so $c \leq 6$. At \$6 both earn \$30.
Allowable increase from \$4: 2

Inside the band the plan does
not change. The profit total does:
 $Z = 2c + 18$ moves with every cent

Taylor 13e, pp. 104 to 108, which calls it the range of optimality. The jar candle's profit is held fixed at \$3 throughout; the report prints one range per variable, each computed with the others still.

Is the accountant's \$3 to \$5 safe?

- The quote sits **inside** the band, so the plan of 2 pillar and 6 jar candles stands across it. No re-planning on a price wiggle.
- Except that **\$3 is right on the boundary**. At exactly \$3 the corners (2, 6) and (0, 8) tie at 24, **multiple optima**, and a cent below it the plan flips to all jar candles.

*A range endpoint is a **knife edge**. Treat a coefficient sitting near one with the caution you would give a number that has already changed.*

What the report prints: the Variable Cells block

Solver's Sensitivity Report for the workshop, one row per product

Variable	Final value	Reduced cost	Coefficient	Increase	Decrease
pillar candles x	2	0	4	2	1
jar candles y	6	0	3	1	1

- The pillar-candle row is today's band: \$4, up 2 to \$6, down 1 to \$3. You do not recompute slopes; you read two columns.
- The jar-candle row is the same question asked of the other coefficient: \$3 holds from \$2 to \$4, with the pillar candle's \$4 held still.

Each range holds the other inputs fixed. An infinite allowable increase would keep the same plan optimal at any higher profit coefficient; it would not make this bounded workshop unbounded.

Where we are

The coefficient band is known; now move a resource



The pillar-candle range is \$3 to \$6, with endpoint ties. Now keep the profit rates fixed and value one extra melter-hour or labor-hour.

Before the practice: one more unit, one re-solve

Change one resource	Best (x, y)	Profit $4x + 3y$	Gain
None: melter 10, labor 8	(2, 6)	26	baseline
Melter 11, labor still 8	(3, 5)	$12 + 15 = 27$	1
Labor 9, melter still 10	(1, 8)	$4 + 24 = 28$	2
Fragrance oil 41, others fixed	(2, 6)	26	0

For the melter change, solve $2x + y = 11$ and $x + y = 8$: subtract to get $x = 3$, then $y = 5$. Check the other corners too.

The added unit is worth the gain, $27 - 26 = 1$, not the new total. This local rate is the shadow price; next check its range.

Week 9 page. Profit and gains in dollars. Each row changes only one resource.

The shadow price, in symbols

The rate at which optimal profit rises per extra unit of a binding resource

$$\text{shadow price} = \frac{\Delta Z^*}{\Delta \text{RHS}}$$

How much the best profit

moves: 10 melter-hours give (2, 6) and \$26; 11 give (3, 5) and \$27. Up by 1

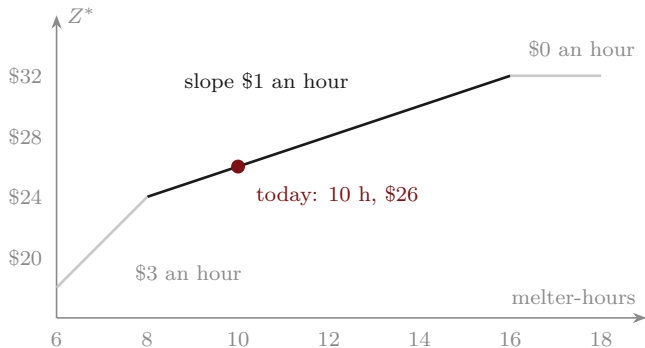
Per unit of the resource: one more hour on the right-hand side of $2x + y \leq 10$

The rate: \$1 a melter-hour. Rent below \$1 is profit, \$1 breaks even, above \$1 loses money

Taylor 13e, pp. 113 to 115: the dollar amount one would be willing to pay for one additional resource unit. Solver's report may say dual price or marginal value; same number.

The shadow price is a slope, and it holds only over a range

Optimal profit as the melter's hours move, labor fixed at 8



From 8 to 16 hours each extra hour adds exactly \$1: the shadow price

Past 16 hours labor caps the plan at 8 pillar candles; an extra hour is worth \$0

Below 8 hours another corner takes over: \$3 an hour. The \$1 holds only from 8 to 16

Taylor 13e, pp. 109 to 113, on the workshop's numbers. The book calls the band the range of feasibility.

What the report prints: the Constraints block

Solver's Sensitivity Report for the workshop, one row per resource

Constraint	Final value	Shadow price	RHS	Increase	Decrease
melter, $2x + y \leq 10$	10	1	10	6	2
labor, $x + y \leq 8$	8	2	8	2	3
fragrance oil, $x + 3y \leq 40$	20	0	40	1E+30	20

The melter's \$1 holds from $10 - 2 = 8$ to $10 + 6 = 16$ hours: the picture's middle segment, printed as two numbers. Labor's \$2 holds from 5 to 10: rent it at \$1.50 and hours 9 and 10 each net \$0.50, and an eleventh is worth \$0.

Binding rows print a price and its range. Fragrance oil prints 0, an increase of 1E+30 (Excel's infinity) and a decrease of 20, its slack.

Taylor 13e, p. 111, Exhibit 3.13. Excel heads the columns Constraint R.H. Side, Allowable Increase, Allowable Decrease; the book says constraint quantity values.

Where we are

Resource values are known; now price the unmade pack



A melter-hour adds \$1 and a labor-hour \$2, within their ranges; fragrance oil has a zero price. These prices also value the resources an unmade product would use.

Why the tea-light pack is not in the plan

Reduced cost: the variable side of the report

- A **tea-light pack** would earn \$2 and use 1 melter-hour and 1 labor-hour; tea lights go unscented, so no fragrance oil. Solver still makes **0 packs**.
- Those resources are worth $1(\$1) + 1(\$2) = \$3$ at the current shadow prices, and a pack returns only \$2. It loses \$1 of value against the pillar and jar candles it would displace: a **reduced cost of $-\$1$** .
- At **\$3** its reduced cost reaches zero: a plan including it can tie for best. The column converts “do not make this” into a price at which the answer changes.

Reduced cost is the mirror of a shadow price: what a sidelined product is short by, per unit.

Beyond the book: Taylor never defines reduced cost; it appears only as a zero-valued column in Exhibits 3.11 and 3.12. The take-home’s decal has a real one.

In practice: marginal, local, and one at a time

IN PRACTICE

One change at a time. The separate ranges do not guarantee that two shadow prices still apply to a joint resource purchase. Check the combined change or re-solve.

Non-binding means \$0, and you can lose its slack for free. More fragrance oil buys nothing; the workshop could also give up its 20 spare ounces before the plan changed.

Some what-ifs break the report. A larger pillar candle (3 melter-hours each) or a brand-new constraint reshapes the region. Re-solve; do not read a range.

A melter-hour priced below \$1 pays for itself. Compare each resource's gain minus its price before buying. Every price on the report is a one-at-a-time, inside-the-range answer, and some what-ifs it cannot answer at all.

The reading's two shapes, both on the workshop

Chapter 4's product mix and diet examples, one shape each

Shape	Variables	Constraints	Objective
Product mix: the workshop, or the take-home's sign shop with three products	what to make: pillar candles, jar candles, a third product	each resource is a $\leq$ limit: melter-hours, labor-hours, fragrance oil	maximize profit
Diet: the workshop blending two bought waxes into its own candle wax	how much of each input to buy: pounds of each wax	each property the blend must have is a $\geq$ floor it must meet	minimize cost

Same Solver, same report. Recognize the shape first: what you choose, what limits or floors it, and which way the objective points.

Taylor 13e, pp. 135 to 143: A Product Mix Example and A Diet Example. The take-home is the first shape; the self-check asks about the second.

Back to the wax price

- The plan **2 pillar candles, 6 jar candles** changes at exactly two pillar-candle profits: below \$3 it jumps to all jar candles, above \$6 to all pillar candles. The \$3 to \$5 quote sits inside.
- So you do **not** re-plan every time the wax price moves. You re-plan only if the pillar candle's profit escapes the range, and you watch the \$3 end, where the plan ties with all jar candles.

One report answers a month of “should we change the plan?” questions.

This week's take-home

One file, on Canvas, by Sunday at 11:59 PM

- A city **sign shop**: street signs, banners and decals against vinyl, press hours and rack space. Formulate it, convert the vinyl from rolls, and **predict** which limits bind before you solve.
- Then Solver, and this time **Reports, Sensitivity Report**: three shadow prices (one of them \$0), the vinyl's range of feasibility, the sign's range of optimality, and the decal's reduced cost. Price each donation at its own shadow price **times the units offered**, stopping at that resource's allowable increase, and the donor question answers itself.

ON THE SITE · the brief and the workbook

Download the workbook here. **Tier 1**: AI as a tool, disclosed.

v348.jima.me/week/w09/



Wednesday is reserved for the midterm

Wed Oct 21: TA-administered setup and testing

2:20 PM

setup begins

2:40 to 3:10

planned test

20 questions

30 minutes, 10 points

Bring your laptop with **Respondus LockDown Browser** installed. The questions cover **Weeks 1 to 8**, with the Canvas calculator, no Excel, notes or AI. Today's Week 9 sensitivity material is outside that pool.

ON THE SITE · exam arrangements

The TA checks submissions and handles technical contingencies through the end of class. You may leave after your submission is verified.

v348.jima.me/exams/



Group practice

IN CLASS

A campus garden grows **tomato boxes** and **herb boxes** at \$6 and \$5 profit, against plot space ($2x + y \leq 12$) and watering time ($x + y \leq 8$); its optimum is 4 and 4 for \$44, both limits binding. It can rent one more square meter **or** one more watering hour. Find the optimal plan with each limit raised by one, compute the gain, and say which resource is worth more and the most to pay for it. **Every member's name goes on the sheet**; one of you photographs it and **uploads the photo to Canvas** before leaving. One participation point, half today's attendance record.

ON THE SITE · Week 9 practice sheet

One extra unit: re-solve and measure the gain.

v348.jima.me/assets/docx/v348-w09-s1-group-practice.docx



Photograph your attempt, then check it together

Two minutes to capture your work; eight for the instructor's demonstration

IN CLASS

Check every member's name and photograph your group's own attempt before the solution is shown. One member uploads the photo to Canvas by the end of class.

We close by checking the method, the main results with units, and a common mistake. Participation credit is for attempting the work.

Session 2

Wed Oct 21

Set up your laptop

2:20 to 2:40 PM; the TA administers today's session

Sign in on the attendance sheet. Close other applications, open **Respondus LockDown Browser**, sign in to Canvas, and locate the midterm.

If you have a technical problem, tell the TA now. Existing accommodation arrangements apply.

ON THE SITE · exam instructions

Today is reserved for setup, the test, and submission verification.

v348.jima.me/exams/



Take the midterm

Planned test: 2:40 to 3:10 PM

20 questions

0.5 point each

30 minutes

10 points total

Weeks 1 to 8

the announced question pool

Answer individually and unaided. Use the Canvas calculator. No Excel, notes or AI.
The TA answers procedural questions only during testing.

Submit the quiz in Canvas, then ask the TA to verify that the submission went through.

Verify your submission

After testing, through 3:35 PM

You may leave after the TA confirms your submission. The remaining class time is for verification and technical contingencies.

Attendance is the sign-in signature and the exam submission. Existing make-up and accommodation arrangements remain in place.

Review

Week 9

Can you define these without looking?

sensitivity analysis	objective coefficient	allowable range	allowable increase, decrease
range of optimality	shadow price	right-hand side	range of feasibility
reduced cost	one at a time	technological coefficient	Sensitivity Report
Variable Cells block	Constraints block	infinite allowable increase	a tie at an endpoint

ON THE SITE · Key Concepts

Most of these have a one-line definition and a worked example behind them. Filter by week; the next two frames define most of them.

v348.jima.me/concepts/



Definitions: the objective side

Sensitivity analysis	How far a model's numbers can move before the answer changes, read off one report without re-solving
Allowable range	The band of an objective coefficient over which the plan stays optimal, though the total moves. Taylor: the range of optimality
Allowable increase, decrease	The two report columns that give the band: coefficient plus increase, coefficient minus decrease
Where the band ends	Where the objective line goes parallel to a binding edge, so a whole edge ties
One at a time	Every range holds the other coefficients and every constraint fixed. Two changes together are not covered
Infinite increase	Raising that coefficient never changes which plan is optimal. Not unbounded profit

Definitions: the resource side

Shadow price	The rise in optimal profit per extra unit of a binding resource: the most you would pay for one more. Also dual price
Range of feasibility	The right-hand-side band over which the shadow price holds; outside it the bottleneck has moved
Non-binding	Slack left over, shadow price \$0, like the workshop's fragrance oil. More of it buys nothing
Reduced cost	For an unmade product, the negative of its profit shortfall. Close that gap and a plan using it can tie for best
Technological coefficient	A per-unit resource use, like 2 melter-hours a pillar candle. Changing one reshapes the region; re-solve
The two blocks	Variable Cells: reduced cost and the objective range. Constraints: shadow price and the RHS range

Test yourself

If one of these is slow, that is where to reread

1. The pillar candle's profit rises from \$4 to \$5.50, inside its \$3 to \$6 range. What happens to the plan, and what happens to the profit total?
2. Why does the range end at exactly \$6, and what is true of the corners $(2, 6)$ and $(5, 0)$ at that value?
3. One more melter-hour is worth \$1 and one more labor-hour \$2. A supplier offers both at once. Why should you check the combined change before using both printed prices?
4. The melter's shadow price holds from 8 to 16 hours. What is an extra hour worth at 17 hours, and why?
5. A tea-light pack earns \$2 and uses one melter-hour and one labor-hour. Compute its reduced cost and the profit at which a plan using it could tie for best.

Numbers worth remembering

\$3 to \$6

pillar-candle range;
(2, 6) holds inside

\$1 and \$2

a melter-hour, a
labor-hour

8 to 16

melter-hours over
which the \$1 holds

-\$1

a tea-light pack's
reduced cost

Question

Workshop

The move

Does the plan survive a new
pillar-candle profit

\$3 to \$6

the objective's slope stays between the
two edges'

What is one more melter-hour
worth

\$1

re-solve at 11 hours: (3, 5), \$27; take the
gain

Why no tea-light packs

short by \$1

\$2 earned against $1(\$1) + 1(\$2)$ of
resources used

Carry into Week 10

- Week 10 gives the linear program **structure**: a transportation grid of plants and stores, a transshipment hub in between, and an assignment of crews to the stores' delivery runs.
- Same Solver, same three cells, but the decision variables sit in a grid and the constraints are one equality per row and per column, and the answers come out in **whole units** without being asked.
- Then the case where they do not: integer programming, and why rounding the fractional answer is not the whole-number answer.

ON THE SITE · Week 10

Two chapters this time, with the software sections skippable. Do the self-check before Monday.

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